SECTION 1

Momentum and Impulse

SECTION OBJECTIVES

- Compare the momentum of different moving objects.
- Compare the momentum of the same object moving with different velocities.
- Identify examples of change in the momentum of an object.
- Describe changes in momentum in terms of force and time.

LINEAR MOMENTUM

When a soccer player heads a moving ball during a game, the ball's velocity changes rapidly. After the ball is struck, the ball's speed and the direction of the ball's motion change. The ball moves across the soccer field with a different speed than it had and in a different direction than it was traveling before the collision.

The quantities and kinematic equations describing one-dimensional motion predict the motion of the ball before and after the ball is struck. The concept of force and Newton's laws can be used to calculate how the motion of the ball changes when the ball is struck. In this chapter, we will examine how the force and the duration of the collision between the ball and the soccer player affect the motion of the ball.

Momentum is mass times velocity

To address such issues, we need a new concept, **momentum.** *Momentum* is a word we use every day in a variety of situations. In physics this word has a specific meaning. The linear momentum of an object of mass m moving with a velocity $\mathbf{v}$ is defined as the product of the mass and the velocity. Momentum is represented by the symbol $\mathbf{p}$.

momentum

a quantity defined as the product of the mass and velocity of an object

MOMENTUM

$$\mathbf{p} = m\mathbf{v}$$

momentum = mass × velocity

As its definition shows, momentum is a vector quantity, with its direction matching that of the velocity. Momentum has dimensions mass × length/time, and its SI units are kilogram-meters per second (kg•m/s).

If you think about some examples of the way the word *momentum* is used in everyday speech, you will see that the physics definition conveys a similar meaning. Imagine coasting down a hill of uniform slope on your bike without pedaling or using the brakes. Because of the force of gravity, you will accelerate; that is, your velocity will increase with time. This idea is often expressed by saying that you are "picking up speed" or "gathering momentum." The faster you move, the more momentum you have and the more difficult it is to come to a stop.

Figure 1
A bicycle rolling downhill has momentum. An increase in either mass or speed will increase the momentum.

Imagine rolling a bowling ball down one lane at a bowling alley and rolling a playground ball down another lane at the same speed. The more massive bowling ball exerts more force on the pins than the playground ball exerts because the bowling ball has more momentum than the playground ball does. When we think of a massive object moving at a high velocity, we often say that the object has a large momentum. A less massive object with the same velocity has a smaller momentum.

On the other hand, a small object moving with a very high velocity may have a larger momentum than a more massive object that is moving slowly does. For example, small hailstones falling from very high clouds can have enough momentum to hurt you or cause serious damage to cars and buildings.

Did you know?

Momentum is so fundamental in Newton's mechanics that Newton called it simply "quantity of motion." The symbol for momentum, **p**, comes from German mathematician Gottfried Leibniz. Leibniz used the term *progress* to mean "the quantity of motion with which a body proceeds in a certain direction."

SAMPLE PROBLEM A

Momentum

PROBLEM

A 2250 kg pickup truck has a velocity of 25 m/s to the east. What is the momentum of the truck?

SOLUTION

Given: $m = 2250$ kg $\mathbf{v} = 25$ m/s to the east

Unknown: $\mathbf{p} = ?$

Use the definition of momentum.

$$\mathbf{p} = m\mathbf{v} = (2250 \text{ kg})(25 \text{ m/s east})$$

$$\boxed{\mathbf{p} = 5.6 \times 10^4 \text{ kg}\bullet\text{m/s to the east}}$$

TIP *Momentum is a vector quantity, so you must specify both its size and direction.*

PRACTICE A

Momentum

1. A deer with a mass of 146 kg is running head-on toward you with a speed of 17 m/s. You are going north. Find the momentum of the deer.
2. A 21 kg child on a 5.9 kg bike is riding with a velocity of 4.5 m/s to the northwest.
   a. What is the total momentum of the child and the bike together?
   b. What is the momentum of the child?
   c. What is the momentum of the bike?
3. What velocity must a 1210 kg car have in order to have the same momentum as the pickup truck in Sample Problem A?

Figure 2
When the ball is moving very fast, the player must exert a large force over a short time to change the ball's momentum and quickly bring the ball to a stop.

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impulse

the product of the force and the time over which the force acts on an object

A change in momentum takes force and time

Figure 2 shows a player stopping a moving soccer ball. In a given time interval, he must exert more force to stop a fast ball than to stop a ball that is moving more slowly. Now imagine a toy truck and a real dump truck rolling across a smooth surface with the same velocity. It would take much more force to stop the massive dump truck than to stop the toy truck in the same time interval. You have probably also noticed that a ball moving very fast stings your hands when you catch it, while a slow-moving ball causes no discomfort when you catch it. The fast ball stings because it exerts more force on your hand than the slow-moving ball does.

From examples like these, we see that a change in momentum is closely related to force. In fact, when Newton first expressed his second law mathematically, he wrote it not as $\mathbf{F} = m\mathbf{a}$, but in the following form.

$$\mathbf{F} = \frac{\Delta \mathbf{p}}{\Delta t}$$

$$\text{force} = \frac{\text{change in momentum}}{\text{time interval}}$$

We can rearrange this equation to find the change in momentum in terms of the net external force and the time interval required to make this change.

IMPULSE-MOMENTUM THEOREM

$$\mathbf{F}\Delta t = \Delta \mathbf{p} \qquad \text{or} \qquad \mathbf{F}\Delta t = \Delta \mathbf{p} = m\mathbf{v_f} - m\mathbf{v_i}$$

force × time interval = change in momentum

This equation states that a net external force, **F,** applied to an object for a certain time interval, Δt, will cause a change in the object's momentum equal to the product of the force and the time interval. In simple terms, a small force acting for a long time can produce the same change in momentum as a large force acting for a short time. In this book, all forces exerted on an object are assumed to be constant unless otherwise stated.

The expression $\mathbf{F}\Delta t = \Delta \mathbf{p}$ is called the impulse-momentum theorem. The term on the left side of the equation, $\mathbf{F}\Delta t$, is called the **impulse** of the force **F** for the time interval Δt.

The equation $\mathbf{F}\Delta t = \Delta \mathbf{p}$ explains why proper technique is important in so many sports, from karate and billiards to softball and croquet. For example, when a batter hits a ball, the ball will experience a greater change in momentum if the batter keeps the bat in contact with the ball for a longer time. Extending the time interval over which a constant force is applied allows a smaller force to cause a greater change in momentum than would result if the force were applied for a very short time. You may have noticed this fact when pushing a full shopping cart or moving furniture.

SAMPLE PROBLEM B

Force and Impulse

PROBLEM

A 1400 kg car moving westward with a velocity of 15 m/s collides with a utility pole and is brought to rest in 0.30 s. Find the force exerted on the car during the collision.

SOLUTION

Given: $m = 1400$ kg $\quad \mathbf{v_i} = 15$ m/s to the west, $v_i = -15$ m/s

$\Delta t = 0.30$ s $\quad \mathbf{v_f} = 0$ m/s

Unknown: $\mathbf{F} = ?$

Use the impulse-momentum theorem.

$$\mathbf{F}\Delta t = \Delta \mathbf{p} = m\mathbf{v_f} - m\mathbf{v_i}$$

$$\mathbf{F} = \frac{m\mathbf{v_f} - m\mathbf{v_i}}{\Delta t}$$

$$F = \frac{(1400 \text{ kg})(0 \text{ m/s}) - (1400 \text{ kg})(-15 \text{ m/s})}{0.30 \text{ s}} = \frac{21\,000 \text{ kg}\bullet\text{m/s}}{0.30 \text{ s}}$$

$$\boxed{\mathbf{F} = 7.0 \times 10^4 \text{ N to the east}}$$

TIP: *Create a simple convention for describing the direction of vectors. For example, always use a negative speed for objects moving west or south and a positive speed for objects moving east or north.*

PRACTICE B

Force and Impulse

1. A 0.50 kg football is thrown with a velocity of 15 m/s to the right. A stationary receiver catches the ball and brings it to rest in 0.020 s. What is the force exerted on the ball by the receiver?

2. An 82 kg man drops from rest on a diving board 3.0 m above the surface of the water and comes to rest 0.55 s after reaching the water. What is the net force on the diver as he is brought to rest?

3. A 0.40 kg soccer ball approaches a player horizontally with a velocity of 18 m/s to the north. The player strikes the ball and causes it to move in the opposite direction with a velocity of 22 m/s. What impulse was delivered to the ball by the player?

4. A 0.50 kg object is at rest. A 3.00 N force to the right acts on the object during a time interval of 1.50 s.

a. What is the velocity of the object at the end of this interval?

b. At the end of this interval, a constant force of 4.00 N to the left is applied for 3.00 s. What is the velocity at the end of the 3.00 s?

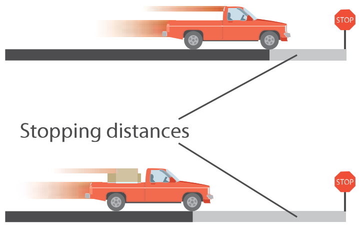

Figure 3
The loaded truck must undergo a greater change in momentum in order to stop than the truck without a load.

Stopping times and distances depend on the impulse-momentum theorem

Highway safety engineers use the impulse-momentum theorem to determine stopping distances and safe following distances for cars and trucks. For example, the truck hauling a load of bricks in **Figure 3** has twice the mass of the other truck, which has no load. Therefore, if both are traveling at 48 km/h, the loaded truck has twice as much momentum as the unloaded truck. If we assume that the brakes on each truck exert about the same force, we find that the stopping time is two times longer for the loaded truck than for the unloaded truck, and the stopping distance for the loaded truck is two times greater than the stopping distance for the truck without a load.

SAMPLE PROBLEM C

Stopping Distance

PROBLEM

A 2240 kg car traveling to the west slows down uniformly from 20.0 m/s to 5.00 m/s. How long does it take the car to decelerate if the force on the car is 8410 N to the east? How far does the car travel during the deceleration?

SOLUTION

Given: $m = 2240$ kg $\quad \mathbf{v_i} = 20.0$ m/s to the west, $v_i = -20.0$ m/s

$\mathbf{v_f} = 5.00$ m/s to the west, $v_f = -5.00$ m/s

$\mathbf{F} = 8410$ N to the east, $F = +8410$ N

Unknown: $\Delta t = ?$ $\quad \Delta\mathbf{x} = ?$

Use the impulse-momentum theorem.

$$\mathbf{F}\Delta t = \Delta\mathbf{p}$$

$$\Delta t = \frac{\Delta\mathbf{p}}{\mathbf{F}} = \frac{m\mathbf{v_f} - m\mathbf{v_i}}{\mathbf{F}}$$

$$\Delta t = \frac{(2240\ \text{kg})(-5.00\ \text{m/s}) - (2240\ \text{kg})(-20.0\ \text{m/s})}{8410\ \text{kg}\bullet\text{m/s}^2}$$

$$\boxed{\Delta t = 4.00\ \text{s}}$$

$$\Delta x = \tfrac{1}{2}(v_i + v_f)\Delta t$$

$$\Delta x = \tfrac{1}{2}(-20.0\ \text{m/s} - 5.00\ \text{m/s})(4.00\ \text{s})$$

$$\boxed{\Delta\mathbf{x} = -50.0\ \text{m} = 50.0\ \text{m to the west}}$$

TIP *For motion in one dimension, take special care to set up the sign of the speed. You can then treat the vectors in the equations of motion as scalars and add direction at the end.*

PRACTICE C

Stopping Distance

1. How long would the car in Sample Problem C take to come to a stop from its initial velocity of 20.0 m/s to the west? How far would the car move before stopping? Assume a constant acceleration.

2. A 2500 kg car traveling to the north is slowed down uniformly from an initial velocity of 20.0 m/s by a 6250 N braking force acting opposite the car's motion. Use the impulse-momentum theorem to answer the following questions:
   a. What is the car's velocity after 2.50 s?
   b. How far does the car move during 2.50 s?
   c. How long does it take the car to come to a complete stop?

3. Assume that the car in Sample Problem C has a mass of 3250 kg.
   a. How much force would be required to cause the same acceleration as in item 1? Use the impulse-momentum theorem.
   b. How far would the car move before stopping? (Use the force found in **a.**)

Force is reduced when the time interval of an impact is increased

The impulse-momentum theorem is used to design safety equipment that reduces the force exerted on the human body during collisions. Examples of this are the nets and giant air mattresses firefighters use to catch people who must jump out of tall burning buildings. The relationship is also used to design sports equipment and games.

Figure 4 shows an Inupiat family playing a traditional game. Common sense tells us that it is much better for the girl to fall onto the outstretched blanket than onto the hard ground. In both cases, however, the change in momentum of the falling girl is exactly the same. The difference is that the blanket "gives way" and extends the time of collision so that the change in the girl's momentum occurs over a longer time interval. A longer time interval requires a smaller force to achieve the same change in the girl's momentum. Therefore, the force exerted on the girl when she lands on the outstretched blanket is less than the force would be if she were to land on the ground.

Figure 4
In this game, the girl is protected from injury because the blanket reduces the force of the collision by allowing it to take place over a longer time interval.

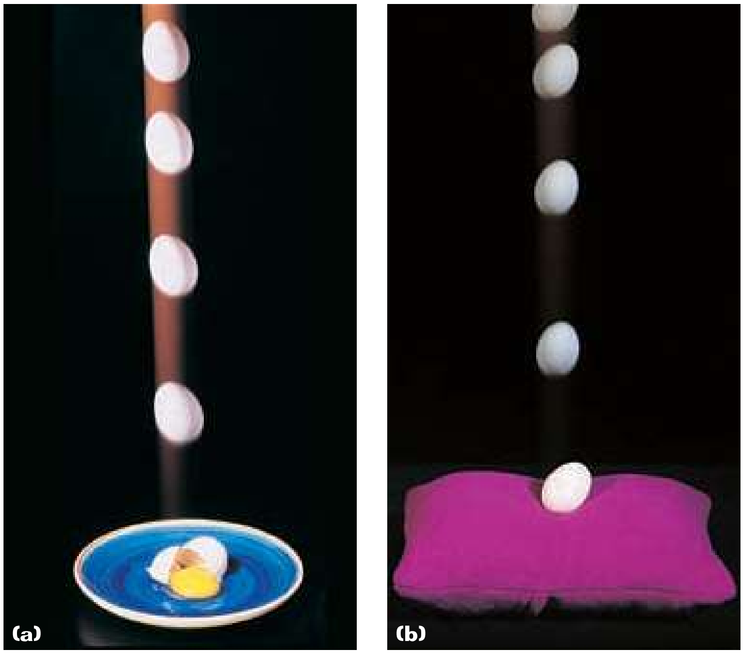

Figure 5
A large force exerted over a short time **(a)** causes the same change in the egg's momentum as a small force exerted over a longer time **(b).**

Now consider a falling egg. When the egg hits a hard surface, like the plate in **Figure 5(a),** the egg comes to rest in a very short time interval. The force the hard plate exerts on the egg due to the collision is large. When the egg hits a floor covered with a pillow, as in **Figure 5(b),** the egg undergoes the same change in momentum, but over a much longer time interval. In this case, the force required to accelerate the egg to rest is much smaller. By applying a small force to the egg over a longer time interval, the pillow causes the same change in the egg's momentum as the hard plate, which applies a large force over a short time interval. Because the force in the second situation is smaller, the egg can withstand it without breaking.

SECTION REVIEW

1. The speed of a particle is doubled.

a. By what factor is its momentum changed?

b. What happens to its kinetic energy?

2. A pitcher claims he can throw a 0.145 kg baseball with as much momentum as a speeding bullet. Assume that a 3.00 g bullet moves at a speed of 1.50×10^3 m/s.

a. What must the baseball's speed be if the pitcher's claim is valid?

b. Which has greater kinetic energy, the ball or the bullet?

3. A 0.42 kg soccer ball is moving downfield with a velocity of 12 m/s. A player kicks the ball so that it has a final velocity of 18 m/s downfield.

a. What is the change in the ball's momentum?

b. Find the constant force exerted by the player's foot on the ball if the two are in contact for 0.020 s.

4. Critical Thinking When a force is exerted on an object, does a large force always produce a larger change in the object's momentum than a smaller force does? Explain.

5. Critical Thinking What is the relationship between impulse and momentum?