SECTION 3

Elastic and Inelastic Collisions

SECTION OBJECTIVES

- **Identify different types of collisions.**
- **Determine the changes in kinetic energy during perfectly inelastic collisions.**
- **Compare conservation of momentum and conservation of kinetic energy in perfectly inelastic and elastic collisions.**
- **Find the final velocity of an object in perfectly inelastic and elastic collisions.**

COLLISIONS

As you go about your day-to-day activities, you probably witness many collisions without really thinking about them. In some collisions, two objects collide and stick together so that they travel together after the impact. An example of this action is a collision between football players during a tackle, as shown in **Figure 10.** In an isolated system, the two football players would both move together after the collision with a momentum equal to the sum of their *momenta* (plural of *momentum*) before the collision. In other collisions, such as a collision between a tennis racquet and a tennis ball, two objects collide and bounce so that they move away with two different velocities.

The total momentum remains constant in any type of collision. However, the total kinetic energy is generally not conserved in a collision because some kinetic energy is converted to internal energy when the objects deform. In this section, we will examine different types of collisions and determine whether kinetic energy is conserved in each type. We will primarily explore two extreme types of collisions: elastic and perfectly inelastic collisions.

Perfectly inelastic collisions can be analyzed in terms of momentum

When two objects, such as the two football players, collide and move together as one mass, the collision is called a **perfectly inelastic collision.** Likewise, if a meteorite collides head on with Earth, it becomes buried in Earth and the collision is perfectly inelastic.

perfectly inelastic collision

a collision in which two objects stick together after colliding

Figure 10
When one football player tackles another, they both continue to fall together. This is one familiar example of a perfectly inelastic collision.

Perfectly inelastic collisions are easy to analyze in terms of momentum because the objects become essentially one object after the collision. The final mass is equal to the combined masses of the colliding objects. The combination moves with a predictable velocity after the collision.

Consider two cars of masses m_1 and m_2 moving with initial velocities of $\mathbf{v_{1,i}}$ and $\mathbf{v_{2,i}}$ along a straight line, as shown in **Figure 11.** The two cars stick together and move with some common velocity, $\mathbf{v_f}$, along the same line of motion after the collision. The total momentum of the two cars before the collision is equal to the total momentum of the two cars after the collision.

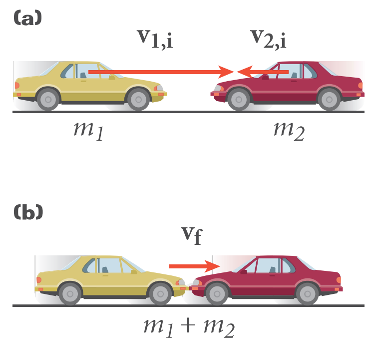

Figure 11
The total momentum of the two cars before the collision **(a)** is the same as the total momentum of the two cars after the inelastic collision **(b).**

PERFECTLY INELASTIC COLLISION

$$m_1\mathbf{v_{1,i}} + m_2\mathbf{v_{2,i}} = (m_1 + m_2)\,\mathbf{v_f}$$

This simplified version of the equation for conservation of momentum is useful in analyzing perfectly inelastic collisions. When using this equation, it is important to pay attention to signs that indicate direction. In **Figure 11,** $\mathbf{v_{1,i}}$ has a positive value (m_1 moving to the right), while $\mathbf{v_{2,i}}$ has a negative value (m_2 moving to the left).

SAMPLE PROBLEM E

Perfectly Inelastic Collisions

PROBLEM

A 1850 kg luxury sedan stopped at a traffic light is struck from the rear by a compact car with a mass of 975 kg. The two cars become entangled as a result of the collision. If the compact car was moving at a velocity of 22.0 m/s to the north before the collision, what is the velocity of the entangled mass after the collision?

SOLUTION

Given: $m_1 = 1850$ kg $m_2 = 975$ kg $\mathbf{v_{1,i}} = 0$ m/s
$\mathbf{v_{2,i}} = 22.0$ m/s to the north

Unknown: $\mathbf{v_f} = ?$

Use the equation for a perfectly inelastic collision.

$$m_1\mathbf{v_{1,i}} + m_2\mathbf{v_{2,i}} = (m_1 + m_2)\,\mathbf{v_f}$$

$$\mathbf{v_f} = \frac{m_1\mathbf{v_{1,i}} + m_2\mathbf{v_{2,i}}}{m_1 + m_2}$$

$$\mathbf{v_f} = \frac{(1850 \text{ kg})(0 \text{ m/s}) + (975 \text{ kg})(22.0 \text{ m/s north})}{1850 \text{ kg} + 975 \text{ kg}}$$

$$\boxed{\mathbf{v_f} = 7.59 \text{ m/s to the north}}$$

PRACTICE E

Perfectly Inelastic Collisions

1. A 1500 kg car traveling at 15.0 m/s to the south collides with a 4500 kg truck that is initially at rest at a stoplight. The car and truck stick together and move together after the collision. What is the final velocity of the two-vehicle mass?

2. A grocery shopper tosses a 9.0 kg bag of rice into a stationary 18.0 kg grocery cart. The bag hits the cart with a horizontal speed of 5.5 m/s toward the front of the cart. What is the final speed of the cart and bag?

3. A 1.50×10^4 kg railroad car moving at 7.00 m/s to the north collides with and sticks to another railroad car of the same mass that is moving in the same direction at 1.50 m/s. What is the velocity of the joined cars after the collision?

4. A dry cleaner throws a 22 kg bag of laundry onto a stationary 9.0 kg cart. The cart and laundry bag begin moving at 3.0 m/s to the right. Find the velocity of the laundry bag before the collision.

5. A 47.4 kg student runs down the sidewalk and jumps with a horizontal speed of 4.20 m/s onto a stationary skateboard. The student and skateboard move down the sidewalk with a speed of 3.95 m/s. Find the following:

a. the mass of the skateboard

b. how fast the student would have to jump to have a final speed of 5.00 m/s

Kinetic energy is not conserved in inelastic collisions

In an inelastic collision, the total kinetic energy does not remain constant when the objects collide and stick together. Some of the kinetic energy is converted to sound energy and internal energy as the objects deform during the collision.

This phenomenon helps make sense of the special use of the words *elastic* and *inelastic* in physics. We normally think of *elastic* as referring to something that always returns to, or keeps, its original shape. In physics, an elastic material is one in which the work done to deform the material during a collision is equal to the work the material does to return to its original shape. During a collision, some of the work done on an *inelastic* material is converted to other forms of energy, such as heat and sound.

The decrease in the total kinetic energy during an inelastic collision can be calculated by using the formula for kinetic energy, as shown in Sample Problem F. It is important to remember that not all of the initial kinetic energy is necessarily lost in a perfectly inelastic collision.

SAMPLE PROBLEM F

Kinetic Energy in Perfectly Inelastic Collisions

PROBLEM

Two clay balls collide head-on in a perfectly inelastic collision. The first ball has a mass of 0.500 kg and an initial velocity of 4.00 m/s to the right. The second ball has a mass of 0.250 kg and an initial velocity of 3.00 m/s to the left. What is the decrease in kinetic energy during the collision?

SOLUTION

1. DEFINE

Given: $m_1 = 0.500$ kg $\quad m_2 = 0.250$ kg

$\mathbf{v_{1,i}} = 4.00$ m/s to the right, $v_{1,i} = +4.00$ m/s

$\mathbf{v_{2,i}} = 3.00$ m/s to the left, $v_{2,i} = -3.00$ m/s

Unknown: $\Delta KE = ?$

2. PLAN

Choose an equation or situation: The change in kinetic energy is simply the initial kinetic energy subtracted from the final kinetic energy.

$$\Delta KE = KE_f - KE_i$$

Determine both the initial and final kinetic energy.

$$\text{Initial:} \quad KE_i = KE_{1,i} + KE_{2,i} = \tfrac{1}{2}m_1 v_{1,i}^2 + \tfrac{1}{2}m_2 v_{2,i}^2$$

$$\text{Final:} \quad KE_f = KE_{1,f} + KE_{2,f} = \tfrac{1}{2}(m_1 + m_2)v_f^2$$

As you did in Sample Problem E, use the equation for a perfectly inelastic collision to calculate the final velocity.

$$\mathbf{v_f} = \frac{m_1\mathbf{v_{1,i}} + m_2\mathbf{v_{2,i}}}{m_1 + m_2}$$

3. CALCULATE

Substitute the values into the equation and solve: First, calculate the final velocity, which will be used in the final kinetic energy equation.

$$v_f = \frac{(0.500 \text{ kg})(4.00 \text{ m/s}) + (0.250 \text{ kg})(-3.00 \text{ m/s})}{0.500 \text{ kg} + 0.250 \text{ kg}}$$

$$\mathbf{v_f} = 1.67 \text{ m/s to the right}$$

Next calculate the initial and final kinetic energy.

$$KE_i = \tfrac{1}{2}(0.500 \text{ kg})(4.00 \text{ m/s})^2 + \tfrac{1}{2}(0.250 \text{ kg})(-3.00 \text{ m/s})^2 = 5.12 \text{ J}$$

$$KE_f = \tfrac{1}{2}(0.500 \text{ kg} + 0.250 \text{ kg})(1.67 \text{ m/s})^2 = 1.05 \text{ J}$$

Finally, calculate the change in kinetic energy.

$$\Delta KE = KE_f - KE_i = 1.05 \text{ J} - 5.12 \text{ J}$$

$$\boxed{\Delta KE = -4.07 \text{ J}}$$

4. EVALUATE

The negative sign indicates that kinetic energy is lost.

PRACTICE F

Kinetic Energy in Perfectly Inelastic Collisions

1. A 0.25 kg arrow with a velocity of 12 m/s to the west strikes and pierces the center of a 6.8 kg target.
   a. What is the final velocity of the combined mass?
   b. What is the decrease in kinetic energy during the collision?
2. During practice, a student kicks a 0.40 kg soccer ball with a velocity of 8.5 m/s to the south into a 0.15 kg bucket lying on its side. The bucket travels with the ball after the collision.
   a. What is the final velocity of the combined mass?
   b. What is the decrease in kinetic energy during the collision?
3. A 56 kg ice skater traveling at 4.0 m/s to the north meets and joins hands with a 65 kg skater traveling at 12.0 m/s in the opposite direction. Without rotating, the two skaters continue skating together with joined hands.
   a. What is the final velocity of the two skaters?
   b. What is the decrease in kinetic energy during the collision?

ELASTIC COLLISIONS

When a player kicks a soccer ball, the collision between the ball and the player's foot is much closer to elastic than the collisions we have studied so far. In this case, *elastic* means that the ball and the player's foot remain separate after the collision.

elastic collision

a collision in which the total momentum and the total kinetic energy are conserved

In an **elastic collision,** two objects collide and return to their original shapes with no loss of total kinetic energy. After the collision, the two objects move separately. In an elastic collision, both the total momentum and the total kinetic energy are conserved.

Most collisions are neither elastic nor perfectly inelastic

In the everyday world, most collisions are not perfectly inelastic. That is, colliding objects do not usually stick together and continue to move as one object. Most collisions are not elastic, either. Even *nearly* elastic collisions, such as those between billiard balls or between a football player's foot and the ball, result in some decrease in kinetic energy. For example, a football deforms when it is kicked. During this deformation, some of the kinetic energy is converted to internal elastic potential energy. In most collisions, some of the kinetic energy is also converted into sound, such as the click of billiard balls colliding. In fact, any collision that produces sound is not elastic; the sound signifies a decrease in kinetic energy.

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Elastic and perfectly inelastic collisions are limiting cases; most collisions actually fall into a category between these two extremes. In this third category of collisions, called *inelastic collisions,* the colliding objects bounce and move separately after the collision, but the total kinetic energy decreases in the collision. *For the problems in this book, we will consider all collisions in which the objects do not stick together to be elastic collisions.* Therefore, we will assume that the total momentum and the total kinetic energy each will stay the same before and after a collision in all collisions that are not perfectly inelastic.

Kinetic energy is conserved in elastic collisions

Figure 12 shows an elastic head-on collision between two soccer balls of equal mass. Assume, as in earlier examples, that the balls are isolated on a frictionless surface and that they do not rotate. The first ball is moving to the right when it collides with the second ball, which is moving to the left. When considered as a whole, the entire system has momentum to the left.

After the elastic collision, the first ball moves to the left and the second ball moves to the right. The magnitude of the momentum of the first ball, which is now moving to the left, is greater than the magnitude of the momentum of the second ball, which is now moving to the right. The entire system still has momentum to the left, just as before the collision.

Another example of a nearly elastic collision is the collision between a golf ball and a club. After a golf club strikes a stationary golf ball, the golf ball moves at a very high speed in the same direction as the golf club. The golf club continues to move in the same direction, but its velocity decreases so that the momentum lost by the golf club is equal to and opposite the momentum gained by the golf ball. *The total momentum is always constant throughout the collision. In addition, if the collision is perfectly elastic, the value of the total kinetic energy after the collision is equal to the value before the collision.*

MOMENTUM AND KINETIC ENERGY ARE CONSERVED IN AN ELASTIC COLLISION

$$m_1\mathbf{v}_{1,i} + m_2\mathbf{v}_{2,i} = m_1\mathbf{v}_{1,f} + m_2\mathbf{v}_{2,f}$$

$$\frac{1}{2}m_1 v_{1,i}^2 + \frac{1}{2}m_2 v_{2,i}^2 = \frac{1}{2}m_1 v_{1,f}^2 + \frac{1}{2}m_2 v_{2,f}^2$$

Remember that v is positive if an object moves to the right and negative if it moves to the left.

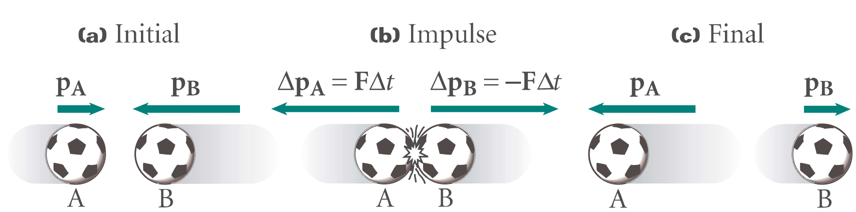


Figure 12
In an elastic collision like this one **(b),** both objects return to their original shapes and move separately after the collision **(c).**

Quick Lab

Elastic and Inelastic Collisions

MATERIALS LIST

- **2 or 3 small balls of different types**

SAFETY

Perform this lab in an open space, preferably outdoors, away from furniture and other people.

Drop one of the balls from shoulder height onto a hard-surfaced floor or sidewalk. Observe the motion of the ball before and after it collides with the ground. Next, throw the ball down from the same height. Perform several trials, giving the ball a different velocity each time. Repeat with the other balls.

During each trial, observe the height to which the ball bounces. Rate the collisions from most nearly elastic to most inelastic. Describe what evidence you have for or against conservation of kinetic energy and conservation of momentum for each collision. Based on your observations, do you think the equation for elastic collisions is useful to make predictions?

SAMPLE PROBLEM G

Elastic Collisions

PROBLEM

A 0.015 kg marble moving to the right at 0.225 m/s makes an elastic head-on collision with a 0.030 kg shooter marble moving to the left at 0.180 m/s. After the collision, the smaller marble moves to the left at 0.315 m/s. Assume that neither marble rotates before or after the collision and that both marbles are moving on a frictionless surface. What is the velocity of the 0.030 kg marble after the collision?

SOLUTION

1. DEFINE

Given: $m_1 = 0.015$ kg $\qquad m_2 = 0.030$ kg

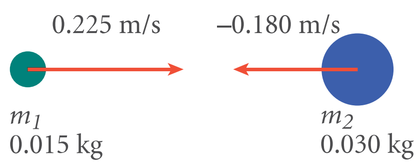
$\mathbf{v_{1,i}} = 0.225$ m/s to the right, $v_{1,i} = +0.225$ m/s

$\mathbf{v_{2,i}} = 0.180$ m/s to the left, $v_{2,i} = -0.180$ m/s

$\mathbf{v_{1,f}} = 0.315$ m/s to the left, $v_{1,f} = -0.315$ m/s

Unknown: $\mathbf{v_{2,f}} = ?$

Diagram:



2. PLAN

Choose an equation or situation: Use the equation for the conservation of momentum to find the final velocity of m_2, the 0.030 kg marble.

$$m_1\mathbf{v_{1,i}} + m_2\mathbf{v_{2,i}} = m_1\mathbf{v_{1,f}} + m_2\mathbf{v_{2,f}}$$

Rearrange the equation to isolate the final velocity of m_2.

$$m_2\mathbf{v_{2,f}} = m_1\mathbf{v_{1,i}} + m_2\mathbf{v_{2,i}} - m_1\mathbf{v_{1,f}}$$

$$\mathbf{v_{2,f}} = \frac{m_1\mathbf{v_{1,i}} + m_2\mathbf{v_{2,i}} - m_1\mathbf{v_{1,f}}}{m_2}$$

3. CALCULATE

Substitute the values into the equation and solve: The rearranged conservation-of-momentum equation will allow you to isolate and solve for the final velocity.

$$v_{2,f} = \frac{(0.015 \text{ kg})(0.225 \text{ m/s}) + (0.030 \text{ kg})(-0.180 \text{ m/s}) - (0.015 \text{ kg})(-0.315 \text{ m/s})}{0.030 \text{ kg}}$$

$$v_{2,f} = \frac{(3.4 \times 10^{-3} \text{ kg}\bullet\text{m/s}) + (-5.4 \times 10^{-3} \text{ kg}\bullet\text{m/s}) - (-4.7 \times 10^{-3} \text{ kg}\bullet\text{m/s})}{0.030 \text{ kg}}$$

$$v_{2,f} = \frac{2.7 \times 10^{-3} \text{ kg}\bullet\text{m/s}}{3.0 \times 10^{-2} \text{ kg}}$$

$$\boxed{\mathbf{v_{2,f}} = 9.0 \times 10^{-2} \text{ m/s to the right}}$$

4. EVALUATE Confirm your answer by making sure kinetic energy is also conserved using these values.

Conservation of kinetic energy

$$\frac{1}{2}m_1 v_{1,i}^2 + \frac{1}{2}m_2 v_{2,i}^2 = \frac{1}{2}m_1 v_{1,f}^2 + \frac{1}{2}m_2 v_{2,f}^2$$

$$KE_i = \frac{1}{2}(0.015 \text{ kg})(0.225 \text{ m/s})^2 + \frac{1}{2}(0.030 \text{ kg})(-0.180 \text{ m/s})^2 = 8.7 \times 10^{-4} \text{ kg}\bullet\text{m}^2/\text{s}^2 = 8.7 \times 10^{-4} \text{ J}$$

$$KE_f = \frac{1}{2}(0.015 \text{ kg})(0.315 \text{ m/s})^2 + \frac{1}{2}(0.030 \text{ kg})(0.090 \text{ m/s})^2 = 8.7 \times 10^{-4} \text{ kg}\bullet\text{m}^2/\text{s}^2 = 8.7 \times 10^{-4} \text{ J}$$

Kinetic energy is conserved.

PRACTICE G

Elastic Collisions

1. A 0.015 kg marble sliding to the right at 22.5 cm/s on a frictionless surface makes an elastic head-on collision with a 0.015 kg marble moving to the left at 18.0 cm/s. After the collision, the first marble moves to the left at 18.0 cm/s.

a. Find the velocity of the second marble after the collision.

b. Verify your answer by calculating the total kinetic energy before and after the collision.

2. A 16.0 kg canoe moving to the left at 12.5 m/s makes an elastic head-on collision with a 14.0 kg raft moving to the right at 16.0 m/s. After the collision, the raft moves to the left at 14.4 m/s. Disregard any effects of the water.

a. Find the velocity of the canoe after the collision.

b. Verify your answer by calculating the total kinetic energy before and after the collision.

3. A 4.0 kg bowling ball sliding to the right at 8.0 m/s has an elastic head-on collision with another 4.0 kg bowling ball initially at rest. The first ball stops after the collision.

a. Find the velocity of the second ball after the collision.

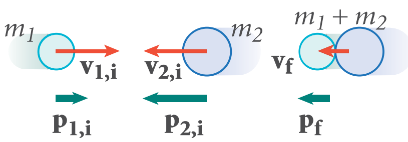
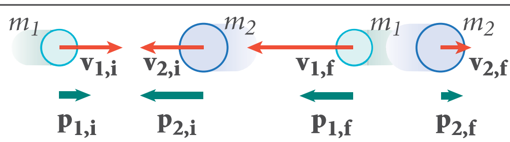
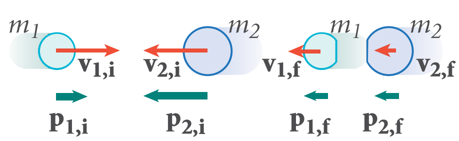
b. Verify your answer by calculating the total kinetic energy before and after the collision.

4. A 25.0 kg bumper car moving to the right at 5.00 m/s overtakes and collides elastically with a 35.0 kg bumper car moving to the right. After the collision, the 25.0 kg bumper car slows to 1.50 m/s to the right, and the 35.0 kg car moves at 4.50 m/s to the right.

a. Find the velocity of the 35 kg bumper car before the collision.

b. Verify your answer by calculating the total kinetic energy before and after the collision.

Table 2 Types of Collisions

Type of collision	Diagram	What happens	Conserved quantity
perfectly inelastic		The two objects stick together after the collision so that their final velocities are the same.	momentum
elastic		The two objects bounce after the collision so that they move separately.	momentum kinetic energy
inelastic		The two objects deform during the collision so that the total kinetic energy decreases, but the objects move separately after the collision.	momentum

SECTION REVIEW

1. Give two examples of elastic collisions and two examples of perfectly inelastic collisions.

2. A 95.0 kg fullback moving south with a speed of 5.0 m/s has a perfectly inelastic collision with a 90.0 kg opponent running north at 3.0 m/s.
   a. Calculate the velocity of the players just after the tackle.
   b. Calculate the decrease in total kinetic energy as a result of the collision.

3. Two 0.40 kg soccer balls collide elastically in a head-on collision. The first ball starts at rest, and the second ball has a speed of 3.5 m/s. After the collision, the second ball is at rest.
   a. What is the final speed of the first ball?
   b. What is the kinetic energy of the first ball before the collision?
   c. What is the kinetic energy of the second ball after the collision?

4. **Critical Thinking** If two automobiles collide, they usually do not stick together. Does this mean the collision is elastic?

5. **Critical Thinking** A rubber ball collides elastically with the sidewalk.
   a. Does each object have the same kinetic energy after the collision as it had before the collision? Explain.
   b. Does each object have the same momentum after the collision as it had before the collision? Explain.